

Solar Neutrino Observables Sensitive to Matter Effects

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Abstract

We discuss constraints on the coefficient A_{MSW} which is introduced to simulate the effect of weaker or stronger matter potential for electron neutrinos with the current and future solar neutrino data. The currently available solar neutrino data leads to a bound $A_{MSW} = 1.47_{+0.54}^{-0.42}({}_{+1.88}^{-0.82})$ at 1σ (3σ) CL, which is consistent with the Standard Model prediction $A_{MSW} = 1$. For weaker matter potential ($A_{MSW} < 1$), the constraint which comes from the flat ${}^8\text{B}$ neutrino spectrum is already very tight, indicating the evidence for matter effects. Whereas for stronger matter potential ($A_{MSW} > 1$), the bound is milder and is dominated by the day-night asymmetry of ${}^8\text{B}$ neutrino flux recently observed by Super-Kamiokande. Among the list of observable of ongoing and future solar neutrino experiments, we find that (1) an improved precision of the day-night asymmetry of ${}^8\text{B}$ neutrinos, (2) precision measurements of the low energy quasi-monoenergetic neutrinos, and (3) the detection of the upturn of the ${}^8\text{B}$ neutrino spectrum at low energies, are the best choices to improve the bound on A_{MSW} .

0.1 Introduction

Neutrino propagation in matter is described by the Mikheyev-Smirnov-Wolfenstein (MSW) theory [1]. It was successfully applied to solve the solar neutrino problem [2], the discrepancy between the data [3, 4, 5, 6, 7] and the theoretical prediction of solar neutrino flux [8], which blossomed into the solution of the puzzle, the large-mixing-angle (LMA) MSW solution. The solution is in perfect agreement with the result obtained by KamLAND [9] which measured antineutrinos from nuclear reactors traversed essentially in vacuum.

The MSW theory relies on neutrino interaction with matter dictated by the standard electroweak theory and the standard treatment of refraction which is well founded in the theory of refraction of light. Therefore, it is believed to be on a firm basis. On the observational side it predicts a severer reduction of the solar neutrino flux at high energies due to the adiabatic transition in matter than at

low energies where the vacuum oscillation effect dominates. Globally, the behavior is indeed seen in the experiments observing ^8B solar neutrinos at high energies [5, 6] and in radiochemical experiments detecting low energy pp and ^7Be neutrinos [3, 4], and more recently by the direct measurement of ^7Be neutrinos by Borexino [7]. For a summary plot of the current status of high and low energy solar neutrinos, see the review of solar neutrinos in this series. Therefore, one can say that the MSW theory is successfully confronted with the available experimental data.

Nevertheless, we believe that further test of the MSW theory is worth pursuing. First of all, it is testing the charged current (CC) contribution to the index of refraction of neutrinos of the Standard Model, which could not be tested anywhere else. Furthermore, in solar neutrinos, the transition from low to high energy behaviors mentioned above has not been clearly seen in a single experiment in a solar model independent manner. The Borexino and KamLAND experiments tried to fill the gap by observing ^8B neutrinos at relatively low energies [10, 11]. SNO made an attempt to lower the threshold energy to 3.5 MeV [12, 13] and the similar challenge is being undertaken by the Super-Kamiokande (SK) group [14]. Obviously, it is of interest to know to what extent these measurements make the test of the MSW theory stringent. Furthermore, the MSW theory is usually assumed to disentangle the genuine effect of CP phase δ from the matter effect in future experiments to determine δ and the mass hierarchy. Therefore, to prove the MSW theory to the accuracy required by such measurement is highly desirable to make these measurement robust. This reasoning has been spelled out in [15].

In this paper we discuss the question of to what extent tests of the MSW theory can be made stringent by various solar neutrino observables. We do this in the light of the past, ongoing and future experiments of solar neutrinos, anticipating the soon coming precision measurements. We adopt the framework for testing the MSW theory by using the measure A_{MSW} [16], defined as the ratio of the effective coupling of weak interactions measured with coherent neutrino matter interactions in the forward direction to the Fermi coupling constant G_F .¹ With various new experimental inputs, it is now quite timely to update the constraint on A_{MSW} which was obtained in [16]. The new data include ^7Be and pep measurement by Borexino [7, 18], the SNO low threshold data [12, 13], Borexino and KamLAND measurement of ^8B flux [10, 11], and most importantly, recent measurement of the day-night asymmetry by SK [14]. Moreover, we give qualitative understandings of the origin of the bound on A_{MSW} from above and below unity. We then discuss how far we can go with a wealth of further solar neutrino observables.

0.2 Simple analytic treatment of matter effect dependences

In this section, we give a simple analytic description of how various solar neutrino observables depend upon the matter effect. It should serve for intuitive understanding of the characteristic features which we will see in the later sections. The reader will find a physics discussion in the flavor conversion review of this series. In the following, we denote the matter densities inside the Sun and in the Earth as ρ_S and ρ_E , respectively. Solar neutrino survival or appearance probabilities depend on three oscillation parameters: the solar oscillation parameters (θ_{12} , $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$), and θ_{13} . Smallness of the

¹ Though the test involves the solar or the Earth electron number densities calculated by the standard solar model (SSM), or by the Preliminary Reference Earth Model (PREM) [17], respectively, we take the attitude that it primarily tests the standard electroweak theory charged current (CC) coupling constant in the forward direction. It is because the solar matter density calculated by SSM is cross checked by helioseismology to an accuracy much better than the one presented here. It is also due to that the Earth matter dependent observable, the day-night variation of solar neutrino flux, is insensitive to the precise profile of the Earth matter density.

recently measured value of θ_{13} [19, 20, 21, 22, 23] and its small error greatly restricts the uncertainty introduced by this parameter on the determination of matter effects.

To quantify possible deviation from the MSW theory, following [16], we introduce the parameter A_{MSW} by replacing the Fermi coupling constant G_F by $A_{MSW}G_F$. The underlying assumption behind such simplified framework is that the deviation from the Fermi coupling constant is universal over fermions, in particular up and down quarks.

The survival probability in the absence of the Earth matter effect, i.e., during the day, is well described by [24, 25, 26]

$$P_{ee}^D = \cos^4 \theta_{13} \left(\frac{1}{2} + \frac{1}{2} \cdot \cos 2\theta_S \cdot \cos 2\theta_{12} \right) + \sin^4 \theta_{13} \quad (1)$$

Here θ_S is the mixing angle at the production point inside the Sun:

$$\cos 2\theta_S \equiv \cos 2\theta_m(\rho_S) \quad (2)$$

where $\theta_m(\rho)$ is the mixing angle in matter of density ρ_S ,

$$\cos 2\theta_S = \frac{\cos 2\theta_{12} - \xi_S}{(1 - 2\xi_S \cos 2\theta_{12} + \xi_S^2)^{1/2}}. \quad (3)$$

In (4), ξ_S is defined as the ratio of the neutrino oscillation length in vacuum, l_ν , to the refraction length in matter, l_0 :

$$\begin{aligned} \xi_S \equiv \frac{l_\nu}{l_0} &= \frac{2\sqrt{2}A_{MSW}G_F\rho_S Y_e \cos^2 \theta_{13}}{m_N} \frac{E}{\Delta m^2} \\ &= 0.203 \times \cos^2 \theta_{13} \left(\frac{E}{1 \text{ MeV}} \right) \left(\frac{\rho_S Y_e}{100 \text{ g cm}^{-3}} \right), \end{aligned} \quad (4)$$

where

$$l_\nu \equiv \frac{4\pi E}{\Delta m^2}, \quad l_0 \equiv \frac{2\pi m_N}{\sqrt{2}A_{MSW}G_F\rho_S Y_e \cos^2 \theta_{13}}. \quad (5)$$

In (4) and (5), ρ_S is the matter density, Y_e is the number of electrons per nucleon, and m_N is the nucleon mass. In the last term we have used the best fit of the global analysis $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{ eV}^2$. The average electron number densities $\rho_S Y_e$ at the production point of various solar neutrino fluxes are tabulated in Table 1.

0.2.1 Energy spectrum

Solar neutrino observables taken in a single experiment have not shown an energy dependence yet. The neutrino oscillation parameters are such that we can not expect strong energy dependences. At low neutrino energies, small ξ_S , Eq. (1) can be approximated by

$$P_{ee}^D = \cos^4 \theta_{13} \left[1 - \frac{1}{2} \sin^2 2\theta_{12} (1 + \cos 2\theta_{12} \xi_S) \right] + \sin^4 \theta_{13} \quad (6)$$

Table 1: Average electron density at the neutrino production region and energy of the relevant pp solar neutrinos fluxes. Last column shows the ratio of the electron neutrino elastic scattering with electrons cross section to the μ (or τ) neutrino one. For this calculation, we have assumed a measured electron kinetic energy range of [0.05,0.4], [1,1.4], [0,0.8], and [5,16] MeV for the pp , pep , ${}^7\text{Be}$ and ${}^8\text{B}$ respectively.

Source	$\rho_S Y_e (\text{g cm}^{-3})$	Energy (MeV)	$\frac{\sigma_\mu}{\sigma_e}$
pp	67.9	≤ 0.42	0.284
pep	73.8	1.44	0.203
${}^7\text{Be}$	86.5	0.86	0.221
${}^8\text{B}$	92.5	≤ 16	0.155

Whereas at high energies, small $\frac{1}{\xi_S}$, the oscillation probability (1) can be approximated, keeping only the first energy dependent term as

$$P_{ee}^D = \cos^4 \theta_{13} \left[\sin^2 \theta_{12} + \frac{1}{4} \sin^2 2\theta_{12} \cos 2\theta_{12} \left(\frac{1}{\xi_S} \right)^2 \right] + \sin^4 \theta_{13} \quad (7)$$

Notice that the correction to the asymptotic behavior is linear in A_{MSW} at low energies while it is quadratic in A_{MSW}^{-1} at high energies. It may mean that the energy spectrum at low energies could be more advantageous in tightening up the constraint on A_{MSW} provided that these formulas with leading order corrections are valid.

It is well known that in the LMA MSW mechanism, ${}^8\text{B}$ neutrino spectrum must show an upturn from the asymptotic high energy ($E \gg 10\text{MeV}$) to lower energies. The behavior is described by the correction term in (7) but only at a qualitative level. It indicates that the upturn component in the spectrum is a decreasing function of A_{MSW} . On the other hand, at low energies populated by pp , ${}^7\text{Be}$, and pep neutrinos, the solar neutrino energy spectrum display vacuum averaged oscillations or decoherence, (6). The deviation from this asymptotic low energy limit can be described by the correction term in (6) again at the (better) qualitative level. The term depend upon A_{MSW} linearly so that the correction term is an increasing function of A_{MSW} . Because of the negative sign in the correction term in (6), larger values of A_{MSW} lead to smaller absolute values of P_{ee} in both low and high energy regions.

To see how accurate is the behavior predicted by the above approximate analytic expressions, we have computed numerically (using the PREM profile) the average $\langle [(1 - r_{\mu/e}) P_{ee} + r_{\mu/e}] (E_{e,i}) \rangle$ as a function of electron energy. Here, $\langle O \rangle$ means taking average of P_{ee} over neutrino energies with neutrino fluxes times the differential cross sections integrated over the true electron energy with response function. In the above expression, $r_{\mu/e} \equiv \frac{\sigma_\mu}{\sigma_e}$ with σ_e and σ_μ being the cross sections of $\nu_e e$ and $\nu_\mu e$ scattering, respectively. The computed results confirm qualitatively the behavior discussed above based on our analytic approximations. Thus, the energy spectrum of solar neutrinos at low and high energies can constrain A_{MSW} in this way, as will be shown quantitatively in Sec. 0.3.

0.2.2 Day-night variation

The ν_e survival probability at night during which solar neutrinos pass through the earth can be written, assuming adiabaticity, as [27]

$$P_{ee}^N = P_{ee}^D - \cos 2\theta_S \cos^2 \theta_{13} \langle f_{reg} \rangle_{\text{zenith}} \quad (8)$$

where P_{ee}^D is the one given in (1). f_{reg} denotes the regeneration effect in the earth, and is given as $f_{reg} = P_{2e} - \sin^2 \theta_{12} \cos^2 \theta_{13}$, where P_{2e} is the transition probability of second mass eigenstate to ν_e . Under the constant density approximation in the earth, f_{reg} is given by [27]

$$f_{reg} = \xi_E \cos^2 \theta_{13} \sin^2 2\theta_E \sin^2 \left[A_{MSW} a_E \cos^2 \theta_{13} (1 - 2\xi_E^{-1} \cos^2 \theta_{12} + \xi_E^{-2})^{\frac{1}{2}} \left(\frac{L}{2} \right) \right] \quad (9)$$

for passage of distance L , where we have introduced $a_E \equiv \sqrt{2} G_F N_e^{earth} = \frac{\sqrt{2} G_F \rho_E Y_{eE}}{m_N}$. In (9), θ_E and ξ_E stand for the mixing angle and the ξ parameter [see (4)] with matter density ρ_E in the earth. Within the range of neutrino parameters allowed by the solar neutrino data, the oscillatory term averages to $\frac{1}{2}$ in good approximation when integrated over zenith angle. Then, the equation simplifies to

$$\langle f_{reg} \rangle_{\text{zenith}} = \frac{1}{2} \cos^2 \theta_{13} \xi_E \sin^2 2\theta_E. \quad (10)$$

At $E = 7$ MeV, which is a typical energy for ^8B neutrinos, $\xi_E = 3.98 \times 10^{-2}$ and $\sin 2\theta_E = 0.940$ for the average density $\bar{\rho}_E = 5.6 \text{ g/cm}^3$ and the electron fraction $Y_{eE} = 0.5$ in the Earth. Then, $\langle f_{reg} \rangle_{\text{zenith}}$ is given as $\langle f_{reg} \rangle_{\text{zenith}} = 1.72 \times 10^{-2}$ for $A_{MSW} = 1$ and $\sin^2 2\theta_{13} = 0.089$. This result is in reasonable agreement with more detailed estimate using the PREM profile [17] for the Earth matter density.

We now give a simple estimate of the day-night asymmetry A_{DN} assuming constant matter density approximation in the earth, and its A_{MSW} dependence. Under the approximation of small regeneration effect $f_{reg} \ll 1$, the day-night asymmetry A_{DN} for the CC number of counts N_{CC} measurement is approximately given by

$$A_{DN}^{CC} \equiv \frac{N_{CC}^N - N_{CC}^D}{\frac{1}{2}[N_{CC}^N + N_{CC}^D]} \approx -\frac{2 \cos 2\theta_S}{1 + \cos 2\theta_{12} \cos 2\theta_S} \langle f_{reg} \rangle_{\text{zenith}} \quad (11)$$

where in the right-hand-side we have approximated A_{DN}^{CC} by the asymmetry of survival probabilities in day and in night at an appropriate neutrino energy, and ignored the terms of order $\langle f_{reg} \rangle_{\text{zenith}}^2$. Notice that the effects of the solar and the earth matter densities are contained only in $\cos 2\theta_S$ and $\langle f_{reg} \rangle_{\text{zenith}}$, respectively.

At $E = 7$ MeV, $\xi_S = 1.31$, $\cos 2\theta_{12} = 0.377$, $\cos 2\theta_S = -0.710$, and hence $A_{DN}^{CC} = 3.41 \times 10^{-2} A_{MSW} \cos^4 \theta_{13}$, about 3% day-night asymmetry for $A_{MSW} = 1$. Note that $\cos^4 \theta_{13} = 0.95$ for $\sin^2 2\theta_{13} = 0.1$, so that the impact of θ_{13} on A_{DN}^{CC} give only a minor modification. Though based on crude approximations, the value of A_{DN}^{CC} at $A_{MSW} = 1$ obtained above is in excellent agreement with the one evaluated numerically for SNO CC measurement.

SNO and SK observes the day-night asymmetry by measurement of CC reactions and elastic scattering (CC+NC), respectively. We have computed A_{DN} as a function of A_{MSW} numerically (with PREM profile) without using analytic approximation. The result of A_{DN} scales linearly with A_{MSW} in a good approximation, $A_{DN}^{CC} \approx 0.044 A_{MSW}$. Similarly, the day-night asymmetry for elastic scattering

measurement can be easily computed. Its relationship to the A_{DN}^{CC} can be estimated in the similar manner as in (11),

$$A_{DN}^{ES} \equiv \frac{N_{ES}^N - N_{ES}^D}{\frac{1}{2}[N_{ES}^N + N_{ES}^D]} \approx A_{DN}^{CC} \times \left[1 + \frac{2r_{\mu/e}}{(1 - r_{\mu/e}) [P_{ee}^N + P_{ee}^D]} \right]^{-1}, \quad (12)$$

taking into account the modification due to NC scattering. Using approximate values, $r_{\mu/e} = \frac{1}{6}$ and $\frac{1}{2}[P_{ee}^N + P_{ee}^D] = \frac{1}{3}$ the factor in the square bracket can be estimated to be $\frac{5}{8}$, giving a reasonable approximation for the ratio of A_{DN}^{ES} to A_{DN}^{CC} . A better approximation to the computed results of the A_{MSW} dependence of the asymmetry is given by $A_{DN}^{ES} = 0.02A_{MSW}$.

0.3 Constraints on A_{MSW} by Solar Neutrino Observables

In this section we investigate quantitatively to what extent A_{MSW} can be constrained by the current and the future solar neutrino data. The results of our calculations are presented in Fig. 1, supplemented with the relevant numbers in Table 2. We will discuss the results and its implications to some details in a step-by-step manner. We first discuss the constraints by the data currently available (Sec. 0.3.1). Then, we address the question of how the constraint on A_{MSW} can be tightened with the future solar neutrino data, the spectral upturn of ^8B neutrinos (Sec. 0.3.2), the low energy ^7Be and pep neutrinos (Sec. 0.3.3), and finally the day-night asymmetry of the solar neutrino flux (Sec. 0.3.4). We pay special attention to the question of how the constraints on A_{MSW} depend upon the significance of these measurements.

0.3.1 Current constraint on A_{MSW}

We include in our global analyses the KamLAND and all the available solar neutrino data [3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14]. To obtain all the results quoted in this paper we marginalize over the mixing angles θ_{12} and θ_{13} , the small mass squared difference Δm_{21}^2 , and the solar neutrino fluxes f_i [8, 28] imposing the luminosity constraint [29]. We include in the analysis the θ_{13} dependence derived from the analysis of the atmospheric, accelerator, and reactor data included in Ref. [30] as well as the recent measurement of θ_{13} by [19, 20, 21, 22, 23]. The χ^2 used is defined by

$$\begin{aligned} \chi_{\text{global}}^2(A_{MSW}) &= \text{Marg}[\chi_{\text{solar}}^2(\Delta m_{21}^2, \theta_{12}, \theta_{13}, A_{MSW}, f_B, f_{\text{Be}}, f_{pp}, f_{\text{CNO}}) \\ &+ \chi_{\text{KamLAND}}^2(\Delta m_{21}^2, \theta_{12}, \theta_{13}) + \chi_{\text{REACTOR+ATM+ACC}}^2(\theta_{13})], \end{aligned} \quad (13)$$

where *Marg* implies to marginalize over the parameters shown but not over A_{MSW} . Further details of the analysis methods can be found in Ref. [28].

The currently available neutrino data (blue solid line), which include SNO lower energy threshold data [12, 13] and SK IV [14], do not allow a very precise determination of the A_{MSW} parameter. A distinctive feature of the $\Delta\chi^2$ parabola shown in Fig. 1 is the asymmetry between the small and large A_{MSW} regions. At $A_{MSW} < 1$ the parabola is already fairly steep, and the “wall” is so stiff that can barely be changed by including the future data. While at $A_{MSW} > 1$ the slope is relatively gentle. More quantitatively, $A_{MSW} = 1.47_{-0.42}^{+0.54}$ (-0.82 $^{+1.88}$) at 1σ (3σ) CL. The best fit point with the present data is significantly larger than unity, $A_{MSW} = 1.47$. It was 1.32 before and have driven to the larger value mostly by the new SK data which indicates a stronger matter effect than those expected by the MSW

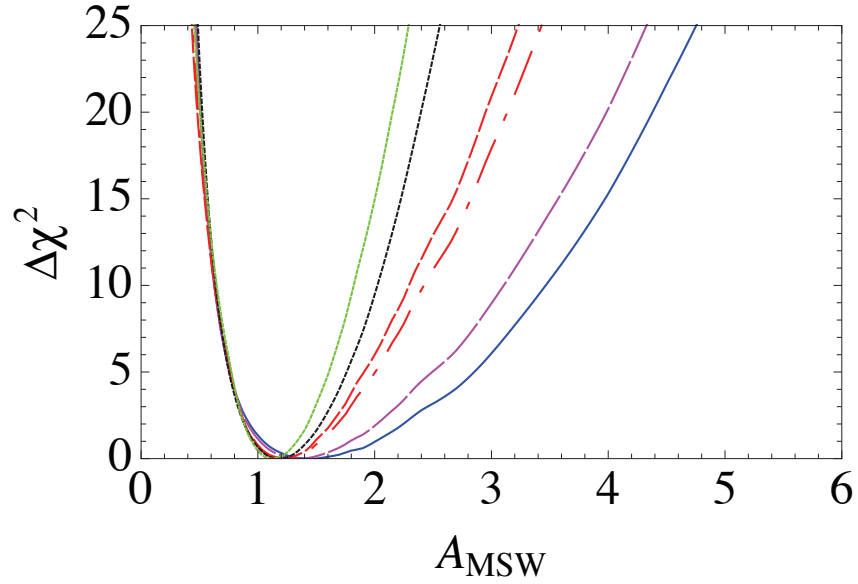


Figure 1: $\Delta\chi^2$ as a function of A_{MSW} for the currently available solar neutrino data (shown in blue solid line) and the various solar neutrino observables expected in the near future (by color lines specified below). The current data include the one from SNO lower energy threshold analysis and SK I-IV. In addition to the current constraints on A_{MSW} , we show the improved constraints when future solar neutrino data are added one by one: 3σ detection of the SK spectral upturn (magenta dashed line), low energy solar neutrino flux measurements of ${}^7\text{Be}$ at 5% and pep at 3% (red dash-dotted line), 3σ detection of the SK day-night asymmetry (black dotted line). The red dashed line shows the improved constraints by adding future spectral information at high and low energies. Finally, the global analysis by adding all the spectral information data and the day-night data produces the solid green line.

Table 2: The $\Delta\chi^2$ minimum of A_{MSW} , the allowed regions of A_{MSW} at 1σ , and 3σ CL are shown in the first, second, and third columns, respectively, for the analyses with the currently available data (first row), the one with spectrum upturn of ^8B neutrinos at 3σ added to the current data (second row), the one with ^7Be and pep neutrinos with 5% and 3% accuracies, respectively, added to the current data (third row), the one with the new spectral information in the second and the third row added to the current data (fourth row), and the one with day-night asymmetry of ^8B neutrinos at 3σ added to the current data (fifth row). The last row presents results of global analysis with all the above data. The numbers in parentheses imply the ones obtained with improved knowledge of θ_{12} , see text for details.

Analysis	$\Delta\chi^2$ minimum	allowed region (1σ)	allowed region (3σ)
present data	$A_{MSW} = 1.47$	1.05–2.01 (1.05–2.00)	0.65–3.35 (0.65–3.27)
+upturn (3σ)	1.34	1.02–1.79 (1.02–1.76)	0.65–3.00 (0.66–2.88)
+ ^7Be (5%), pep (3%)	1.25	0.97–1.53 (0.97–1.52)	0.65–2.34 (0.65–2.31)
+spectral shape	1.22	0.97–1.49 (0.97–1.46)	0.65–2.23 (0.65–2.12)
+ A_{DN} (3σ)	1.17	0.96–1.43 (0.96–1.42)	0.66–1.98 (0.66–1.97)
+all	1.12	0.95–1.33 (0.95–1.32)	0.67–1.78 (0.67–1.73)

LMA region preferred by the KamLAND data. The larger best fit value could also partly be due to an artifact of the weakness of the constraint in $A_{MSW} > 1$ region. Notice that the Standard Model MSW theory value $A_{MSW} = 1$ is off from the 1σ region but only by a tiny amount, as seen in Table 2. Let us understand these characteristics.

The lower bound on A_{MSW} mostly comes from the SK and the SNO data which shows that ^8B neutrino spectrum at high energies is essentially flat with $P_{ee} = \sin^2\theta$, the prediction of the adiabatic LMA MSW solution (see (7)). It is inconsistent with the vacuum oscillation, and hence the point $A_{MSW} = 0$ is highly disfavored, showing the evidence for the matter effect.

One would think that the upper bound on A_{MSW} should come from either the low energy solar neutrino data, or the deviation from the flat spectra at high energies. But, we still lack precise informations on low energy solar neutrinos, and the spectral upturn of ^8B neutrinos has not been observed beyond the level in [10, 11]. Then, what is the origin of the upper bound $A_{MSW} < 2$ at about 1σ CL? We argue that it mainly comes from the day-night asymmetry of ^8B neutrino flux which is contained in the binned data of SK and SNO. Recently, the SK collaboration reported a positive indication of the day-night asymmetry though the data is still consistent with no asymmetry at 2.3σ CL [14].

To show the point, we construct a very simple model for $\Delta\chi^2$ for the day-night asymmetry A_{DN}^{ES} . It is made possible by the approximate linearity of A_{DN}^{ES} to A_{MSW} . Let us start from the data of day-night asymmetry at SK I-IV: $A_{DN}^{ES} = 4.0 \pm 1.3 \pm 0.8\%$ [14], giving the total error 1.5% if added in quadrature. The expectation of A_{DN}^{ES} by the LMA solution is $A_{DN}^{ES} = A_{MSW} \times 2.1\%$ for $\Delta m_{21}^2 = 7.6 \times 10^{-5} \text{ eV}^2$. Then, one can create an approximate model $\Delta\chi^2$ as $\Delta\chi^2 = [(A_{DN} - 2.8\%) / 1.5\%]^2 = 2(A_{MSW} - 1.3)^2$. It explains very well, considering the extremely crude nature of the model, the behavior of $\Delta\chi^2$ with the current data (blue solid line) in Fig. 1 in the region $A_{MSW} > 1$. Therefore, we find that about 2σ evidence of A_{DN}^{ES} in the SK data is the main cause of the sensitivity to A_{MSW}

in the region $A_{MSW} > 1$.

To what extent an improved knowledge of θ_{12} affects A_{MSW} ? It was suggested that a dedicated reactor neutrino experiment can measure $\sin^2 \theta_{12}$ to $\simeq 2\%$ accuracy [31, 32]. It is also expected that precision measurement of pp spectrum could improve the accuracy of θ_{12} determination to a similar extent [28]. Therefore, it is interesting to examine to what extent an improved knowledge of θ_{12} affects the constraint on A_{MSW} . Therefore, we re-compute the $\Delta\chi^2$ curves presented in Fig. 1 by adding the artificial term $(\sin^2 \theta_{12} - BEST)^2/0.02$ in the $\Delta\chi^2$ assuming 2% accuracy in $\sin^2 \theta_{12}$ determination. The result of this computation is given in Table 2 in parentheses. As we see, size of the effect of improved θ_{12} knowledge is not very significant.

0.3.2 Spectrum of solar neutrinos at high energies

We discuss next the impact of observing the spectral upturn of ^8B neutrinos at its relatively low energy part, $E \gtrsim 2 - 3$ MeV. The evidence for the upturn must contribute to constrain the larger values of A_{MSW} because A_{MSW} could be very large without upturn, if day-night asymmetry is ignored. Unfortunately, the upturn has never been observed in the SK, SNO and Borexino measurement; The solar neutrino spectrum is consistent with a flat distribution. We assume that the upturn of ^8B neutrino energy spectrum can soon be established.

We discuss the impact on A_{MSW} of seeing the upturn in recoil electron energy spectrum with 3σ significance, which we assume to be in the region $E_e \geq 3.5$ MeV. To calculate $\Delta\chi^2$ we assume the errors estimated by the SK collaboration [14]. Adding the simulated data to the currently available data set produces the magenta dashed line in Fig. 1. We find a 25% reduction of the 3σ allowed range, $A_{MSW} = 1.34_{+0.45}^{-0.32}({}_{+1.66}^{-0.69})$ at 1σ (3σ) CL. We can see that it does improve the upper bound on A_{MSW} , for which the current constraint (blue solid line) is rather weak, but the improvement in the precision of A_{MSW} is still moderate.

Some remarks are in order about the minimum point of $\Delta\chi^2$. The best fit point with the present data is at $A_{MSW} > 1$ as we saw above. For the analysis with simulated data, the $\Delta\chi^2$ minimum must be always at $A_{MSW} = 1$ in all the cases discussed below. Therefore, the analysis with the present plus simulated data tends to pull the $\Delta\chi^2$ minimum toward smaller values of A_{MSW} , and at the same time make the $\Delta\chi^2$ parabola narrower around the minimum. By conspiracy between these two features the current constraint (blue solid line) is almost degenerate to the other lines at $A_{MSW} < 1$, the ones with spectral upturn (magenta dashed line) and low energy neutrinos (red dash-dotted line). These features can be observed in Fig. 1 and in Table 2.

0.3.3 Spectrum of solar neutrinos at low energies

Now, let us turn to the low energy solar neutrinos. The Borexino collaboration have already measured the ^7Be neutrino-electron scattering rate to an accuracy of $\simeq \pm 5\%$ [7], which we assume throughout this section. However, there is an important limitation on what can be learned from the very precise ^7Be flux. We have to use the SSM flux to determine the neutrino survival probability experimentally, and therefore, the uncertainties in the theoretical estimate [8] limit the precision with which the A_{MSW} parameter can be determined from the ^7Be flux measurement.

The measurement of the pep flux has two important advantages, when compared to the ^7Be flux, in determining A_{MSW} : a) the neutrino energy is higher, 1.44 MeV, so the importance of the solar matter effects is larger, b) the uncertainty in the theoretical estimate is much smaller. Firstly, the ratio of the pep to the pp neutrino flux is robustly determined by the SSM calculations, so it can be

determined more accurately than the individual fluxes because the ratio depends only weakly on the solar astrophysical inputs. Secondly, a very precise measurement of the ${}^7\text{Be}$ flux, with all the other solar data and assuming energy conservation (luminosity constraint), leads to a very precise determination of the pp and pep flux, at the level of $\sim 1\%$ accuracy [28]. See [18] for the first observation of pep neutrinos, and its current status of the uncertainties.

A measurement of the pep neutrino-electron scattering rate must be very accurate, $\simeq \pm 3\%$, in order to significantly constrain our knowledge of the matter effects in the Sun. If this accuracy is achieved, then the uncertainties of the A_{MSW} parameter in region $A_{MSW} > 1$ will be significantly reduced. The red dash-dotted line in Fig. 1 shows the result of the combined analysis of future low energy data, an improved ${}^7\text{Be}$ measurement with 5% precision and a future pep measurement with 3% precision, added to the current data. The obtained constraint on A_{MSW} is: $A_{MSW} = 1.25 \pm 0.28^{(-0.60)}_{(+1.09)}$ at 1σ (3σ) CL. The resultant constraint on A_{MSW} from above is much more powerful than the one obtained with spectrum upturn of high energy ${}^8\text{B}$ neutrinos at 3σ .

By having solar neutrino spectrum informations both at high and low energies it is tempting to ask how tight the constraint become if we combine them. The result of this exercise is plotted by the red dashed line in Fig. 1 and is also given in Table 2. The resultant constraint on A_{MSW} is: $A_{MSW} = 1.22^{(-0.25)}_{(+0.27)}^{(-0.57)}_{(+1.01)}$ at 1σ (3σ) CL.

0.3.4 Day-night asymmetry

To have a feeling on to what extent constraint on A_{MSW} can be tightened by possible future measurement, we extend the simple-minded model discussed in Sec. 0.3.1, but with further simplification of assuming $A_{MSW} = 1$ as the best fit. Let us assume that the day-night asymmetry A_{DN}^{ES} can be determined with $(2/N)\%$ accuracy, an evidence for the day-night asymmetry at $N\sigma$ CL. Then, the appropriate model $\Delta\chi^2$ is given under the same approximations as in Sec. 0.3.1 as $\Delta\chi^2 = N^2 (A_{MSW} - 1)^2$. We boldly assume that the day-night asymmetry at 3σ CL would be a practical goal in SK. It predicts $\Delta\chi^2 = 9(A_{MSW} - 1)^2$, which means that A_{MSW} can be constrained to the accuracy of 33% uncertainty at 1σ CL.

Now, we give the result based on the real simulation of data. The black dotted line in Fig. 1 show the constraint on A_{MSW} obtained by future 3σ CL measurement of the day-night asymmetry, which is added to the present solar neutrino data. As we see, the day-night asymmetry is very sensitive to the matter potential despite our modest assumption of 3σ CL measurement of A_{DN}^{ES} . The obtained constraint on A_{MSW} is: $A_{MSW} = 1.17^{(-0.21)}_{(+0.26)}^{(-0.51)}_{(+0.81)}$ at 1σ (3σ) CL. The obtained upper bound on A_{MSW} is actually stronger than the one expected by our simple-minded model $\Delta\chi^2$. Apart from the shift of the best fit to a larger value of A_{MSW} , the behavior of $\Delta\chi^2$ is more like $\Delta\chi^2 \approx 14(A_{MSW} - 1)^2$ in the region $A_{MSW} > 1$. It can also be seen in Fig. 1 that the upper bound on A_{MSW} due to the day-night asymmetry at 3σ CL (black dotted line) is stronger than the one from combined analysis of all the expected measurements of the shape of the spectrum (red dashed line) discussed at the end of Sec. 0.3.3.

Given the powerfulness of the day-night asymmetry for constraining A_{MSW} , it is highly desirable to measure it at higher CL in the future. Of course, it would be a challenging task, and probably requires a megaton class water Cherenkov or large volume liquid scintillator detectors with solar neutrino detection capability. They include, for example, Hyper-Kamiokande [33], UNO [34], or the ones described in [35].

0.3.5 Global analysis

We now discuss to what extent the constraint on A_{MSW} can become stringent when all the data of various observable are combined. The solid green line in Fig. 1 shows the constraint on A_{MSW} obtained by the global analysis combining all the data sets considered in our analysis. The obtained sensitivity reads $A_{MSW} = 1.12^{+0.17}_{-0.21}({}^{+0.45}_{-0.66})$ at 1σ (3σ) CL. Therefore, the present and the future solar neutrino data, under the assumptions of the accuracies of measurement stated before, can constrain A_{MSW} to $\simeq 15\%$ (40%) at 1σ (3σ) CL from below, and to $\simeq 20\%$ (60%) at 1σ (3σ) CL from above. If we compare this to the current constraint $A_{MSW} = 1.47^{+0.42}_{-0.54}({}^{+0.82}_{-1.88})$ the improvement of the errors for A_{MSW} over the current precision is, very roughly speaking, a factor of $\simeq 1.5 - 2$ in region $A_{MSW} < 1$, and it is a factor of $\simeq 2$ at $A_{MSW} > 1$. Noticing that the efficiency of adding more data to have tighter constraint at $A_{MSW} < 1$ is weakened by shift of the minimum of $\Delta\chi^2$, improvement of the constraint on A_{MSW} is more significant at $A_{MSW} > 1$.

0.4 Summary

In this paper, we have discussed the question of to what extent tests of the MSW theory can be made stringent by various solar neutrino observables. First, we have updated the constraint on A_{MSW} , the ratio of the effective coupling constant of neutrinos to G_F , the Fermi coupling constant with the new data including SNO ${}^8\text{B}$ spectrum and SK day-night asymmetry. Then, we have discussed in detail how and to what extent the solar neutrino observable in the future tighten the constraint on A_{MSW} .

The features of the obtained constraints can be summarized as follows:

- Interpretation of solar neutrino data at high energies by the vacuum oscillation is severely excluded by the SNO and SK experiments, which leads to a strong and robust lower bound of A_{MSW} . On the other hand, the day-night asymmetry at $\simeq 2\sigma$ level observed by SK dominates the bound at high A_{MSW} side. We find that present data lead to $A_{MSW} = 1.47^{+0.42}_{-0.54}({}^{+0.82}_{-1.88})$ at 1σ (3σ) CL. The Standard Model prediction $A_{MSW} = 1$ is outside the 1σ CL range but only by tiny amount.
- We have explored the improvements that could be achieved by solar neutrinos experiments, ongoing and in construction. We discussed three observables that are sensitive enough to significantly improve the limits on A_{MSW} , particularly in the region $A_{MSW} > 1$: a) upturn of the ${}^8\text{B}$ solar neutrino spectra at low energies at 3σ CL, b) high precision measurement of mono-energetic low energy solar neutrinos, ${}^7\text{Be}$ (5% precision) and pep (3% precision) neutrinos, and c) day-night asymmetry of the ${}^8\text{B}$ solar neutrino flux at 3σ CL. They lead to the improvement of the bound as follows:
 - a) $A_{MSW} = 1.34^{+0.32}_{-0.45}({}^{+0.69}_{-1.66})$ at 1σ (3σ) CL.
 - b) $A_{MSW} = 1.25 \pm 0.28({}^{+0.60}_{-1.09})$ at 1σ (3σ) CL.
 - c) $A_{MSW} = 1.17^{+0.21}_{-0.26}({}^{+0.51}_{-0.81})$ at 1σ (3σ) CL.

It could be expected that future measurement by SNO⁺ [36] and KamLAND [37] may detect spectrum modulation of B neutrinos at low energies at CL higher than 3σ .

Finally, by combining all the data set we have considered we obtain $A_{MSW} = 1.11^{+0.18}_{-0.26}({}^{+0.47}_{-0.87})$ at 1σ (3σ) CL.

- In a wider context the constraints on effective neutrino matter coupling has been discussed in the framework of possible nonstandard interactions (NSI denoted as $\varepsilon_{\alpha\beta}$) of neutrinos [38], in

which our A_{MSW} may be interpreted as $A_{MSW} = 1 + \varepsilon_{ee}$ (assuming vanishing of all the other $\varepsilon_{\alpha\beta}$ elements). With solar neutrinos see [39] for discussion of NSI. However, if NSI exist it is likely that the other elements will be detected first in long-baseline experiments; $\varepsilon_{e\mu}$ and $\varepsilon_{e\tau}$ are of second order in ϵ , while ε_{ee} comes in only at third order in ϵ in a perturbative framework [40]. The analyses shows that the sensitivity to ε_{ee} is indeed lower at least by an order of magnitude compared to the ones to $\varepsilon_{e\mu}$ or $\varepsilon_{e\tau}$. See [41] and the references cited therein. Hence, the solar neutrinos are an alternative good probe for ε_{ee} .

In conclusion, testing the theory of neutrino propagation in matter deserves further endeavor. The lack of an accurate measurement of the matter potential felt by solar neutrinos reflects the fact that solar neutrino data only do not precisely determine the mass square splitting. The good match of the independently determined mass square splitting by solar neutrino data and by reactor antineutrino data will confirm the Standard Model prediction of the relative index of refraction of electron neutrinos to the other flavor neutrinos. The lack of match of both measurements would point to new physics like the one tested here.

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